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NUMERICAL METHODS OF COMPUTER OPTICS

PARABOLOIDAL ZONE PLATES: AN EXPERIMENTAL STUDY

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Abstract—Experimental evidence is reported in favour of paraboloidal Fresnel zone plates. Parameters are determined for single- and two-component focusing systems, including frequency properties, longitudinal and transverse resolution, number of image elements in the frame and the field of view for a substantially off-axial source.

It seems to have been Raisky [1] in the early 1950s and Gabor in the mid-1960s [2] who suggested that diffraction objectives might be implemented on surfaces of revolution generated by second-order curves with advantages over planar objectives. Raisky pointed out that the wavefronts for a source at a distance A from a plane zone plate and the observer at B, have opposite curvature, hence the optical path could be incremented by $\lambda/2$ by a rather small increase of the angle between the optical axis and the direction to the nth Fresnel zone. Therefore, for a plane zone plate, the path difference expressed as $\frac{\lambda}{2}$ (number of Fresnel zones) takes on rather large values at comparatively small angles [1].

To obtain high-power diffraction systems having wide Fresnel zones at small apertures and short focal lengths Raisky suggested a spherical surface should be used that provides the same sign of wavefront curvature at A and B. He pointed out also that such a spherical zone plate would be free of spherical aberrations.

Gabor demonstrated later [2] that a Fresnel zone plate could be designed as a spherical element centred at its focal point. Such a zone lens would offer a substantial control of coma-type aberrations and afford beam scans with rather wide angles by displacing the source [2].

A comparative study of the focusing properties of spherical and plane microwave zone-plate antennas has been reported by Dey and Khastgir [3]. They derived a transcendental equation for the radii of Fresnel zones on the zone plate surface. In the microwave range where the wavelength λ is negligible compared with the outer Fresnel zone, the Fresnel zone radii can be determined from the simpler approximate equation

$$\frac{(A+B)^2}{C^2 r^2} R_n^4 + 2\left\{\frac{A+B}{r} + \frac{(A-B)(A+B)^2}{rC^2} - 2\left(\frac{A}{r}+1\right)\right\} R_n^2 + C^2 - 2(A^2+B^2) + (A^2-B^2)^2/C^2 = 0,$$

where $C = A + B + n\lambda/2$, and r is the radius of the antenna sphere.

In the geometrical optics approximation a sensitivity analysis has been carried out to reveal the effect on the focal spot displacement along the optical axis of source mounting at the focus. If a is the new distance from the source to the antenna, then the new focal length b is given according to [3]

$$[(a + r - \sqrt{r^2 - R_1^2})^2 + R_1^2]^{1/2} + [(b - r - \sqrt{r^2 - R_1^2})^2 + R_1^2]^{1/2} - (a + b + \lambda/2) = 0.$$

Analysis of this expression indicates that for $A \gg D$ a variation of source position does not affect the focal length.

A numerical estimate with $a_0 = 60$ cm, $b_0 = 10$ cm, $D = 80$ cm and $\lambda_0 = 3.2$ cm indicates that the spherical diffraction lens offers along the optical axis a somewhat sharper distribution of field intensity than the zone plate. At the same time the side lobes in point imagery are less intense than those of the zone plate; the field intensity at the focus of the spherical diffraction lens being given by the expressions [3]

$$A = I_0 \cdot \sum_{p=0}^{N} -i(\alpha_{2p+1} - \alpha_1),$$

$$\alpha_n = \frac{2\pi}{\lambda}\left[\sqrt{R_n^2 + (a_0 + x_n)^2} + \cdots + \sqrt{R_n^2 - (b - x_n)^2}\right],$$

$$x_n = r - \sqrt{r^2 - R_n^2}.$$

In subsequent work Dey and Khastgir [4] studied a microwave paraboloidal zone plate and Khastgir *et al.* [5] compared the focusing properties of paraboloidal, spherical and plane microwave zone plates. Reference [5] reported expressions for the Fresnel zone radii and demonstrated that the paraboloidal zone plate antenna provides the sharpest intensity distribution at the focus along the optical axis. This property was attributed to the fact that the paraboloidal zone lens accommodates more Fresnel zones in its aperture than the other two. Numerical estimates were derived for the frequency characteristics of a paraboloidal zone plate antenna in the range 8075–10,875 MHz.

A system of equations was presented in [4] to compute the Fresnel zone radii on a paraboloidal zone plate when a spherical wavefront is incident on its convex face.

For a paraboloidal zone plate with $\lambda_0 = 3.2$ cm ($f_0 = 9375$ MHz), $a_0 = 50$ cm, $b_0 = 24$ cm and $N = 12$, the focal length varies from 16 to 34 cm in the above frequency range.

At that time the available evidence on the focusing properties and frequency performance of such lenses lacked the support of appropriate experimental studies. In addition the question of focusing an off-axis source remained unresolved. To fill the gap the present paper reports experimental evidence on the focusing properties and frequency performance of paraboloidal (singlet and doublet) zone plates [6].

The discrete phase function of the lens (Fresnel zones) transforming a diverging spherical wavefront into a converging spherical wavefront is calculated by the expressions [4]

$$\sqrt{R_n^2 + (a + b_n)^2} + \sqrt{(b_0 b_n)^2 + R_n^2} = a + b_0 + n\lambda/2,$$

$$b_n = R_n^2/4F.$$

Consider the resolution of a paraboloidal zone lens. The point spread function of this lens is shown in Fig. 1 as a function of frequency. The solid curve represents the diffraction spot for the zone plate [7].

Analysis of the experimental data plotted in Fig. 1 indicates that:

(1) the paraboloidal zone plate antenna, much like the plane zone plate [7], provides a resolution close to the diffraction limit over a wide range (from 20.65% to −16.3%);

(2) in the distribution of field intensity at the focus, the level of side lobes of paraboloidal zone plates is on average 20–40% below that of plane zone plate antennae.

The distribution of field intensity along the optical axis for three different wavelengths close to the designed λ_0 is shown in Fig. 2. This experimental evidence suggests that the distribution, determined by the two first zeros, is on average 15–20% narrower in width than that of a plane zone plate.

Before we move on to consider the experimental evidence, we should note one important property of zone plates made on second-order surfaces; namely, the principle of reversibility is no longer valid for them. For a plane zone plate, one may interchange the front and back focal lengths. For instance, if a plane zonal plate is designed for a front focal length A and a back focal length B, then by replacing the source at B and the sensing element at A, one retains the properties of the zone plate, because the values of its Fresnel zone radii are independent of A–B permutation. This property of reversibility, however, no longer holds for curved zone plates (diffraction lenses), and one has to pay attention to the orientation of the lens vertex with respect to the design values of A and B in raytracing such designs.

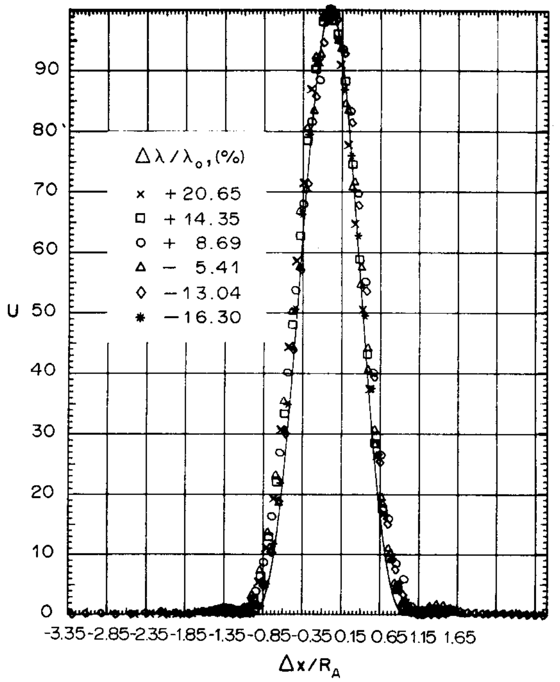

Fig. 1. Diffraction spot for a paraboloidal zone lens.

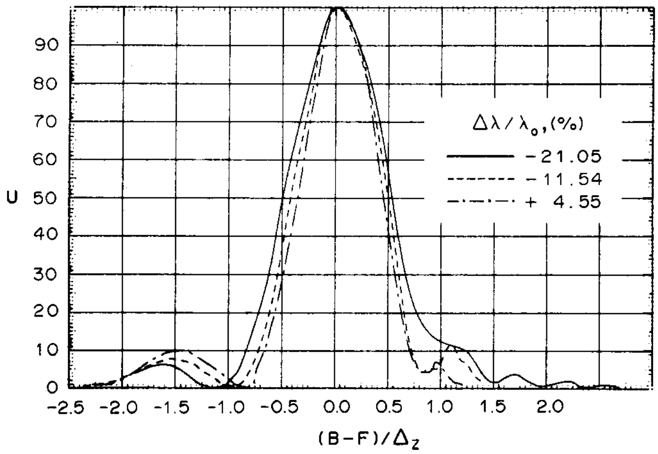

Fig. 2. Distribution of intensity along the optical axis.

PARABOLOIDAL SINGLET

The single-element, paraboloidal, zone-plate antenna shown in Fig. 3 was manufactured from polyethylene on a CNC machine tool. The lens aperture, $D/\lambda_0 \approx 44$; the front focal length, $A/\lambda_0 \approx 87$; the back focal length, $B/\lambda_0 \approx 43$; and the focal length of the paraboloid, $F/\lambda_0 \approx 14$.

The frequency characteristics of the paraboloidal zone lens antennae are plotted in Fig. 4. The plots coincide well in the range of negative wavelength deviations from λ_0 and differ by a mere 3–4% in the range of positive detuning. That the B–λ curves for the plane and paraboloidal zone plates coincide may be attributed to the fact that the surface of a paraboloidal zone lens designed for the above parameters accommodates about the same number of Fresnel zones as the plane zone plate.

Experiments were carried out to examine the resolving properties of the paraboloidal lens for an off-axis source placed at the edge of the field of view. The spread of the main lobe, obtained by the first minima of the diffraction spot, amounts to about 4%, and the intensity of the side

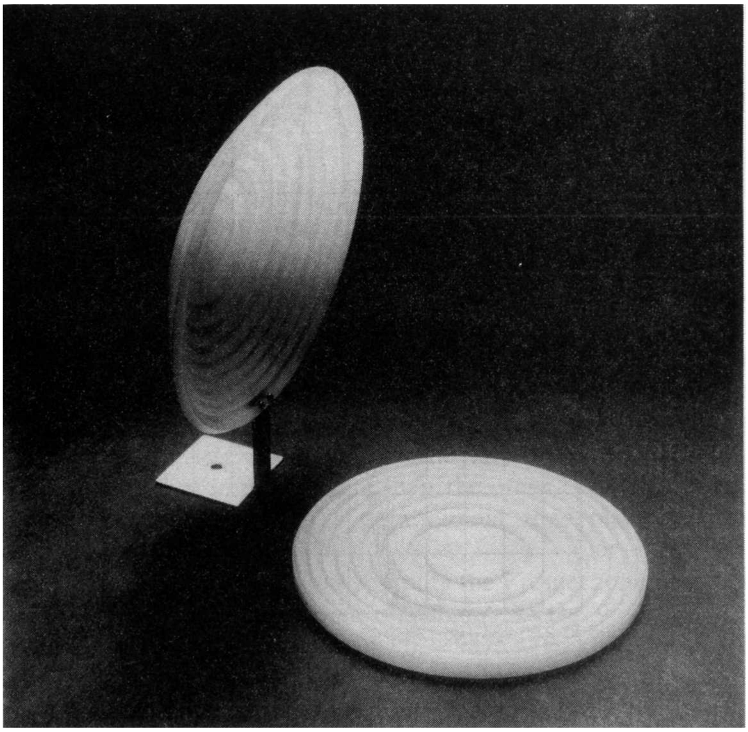

Fig. 3. Paraboloidal zone plate antenna.

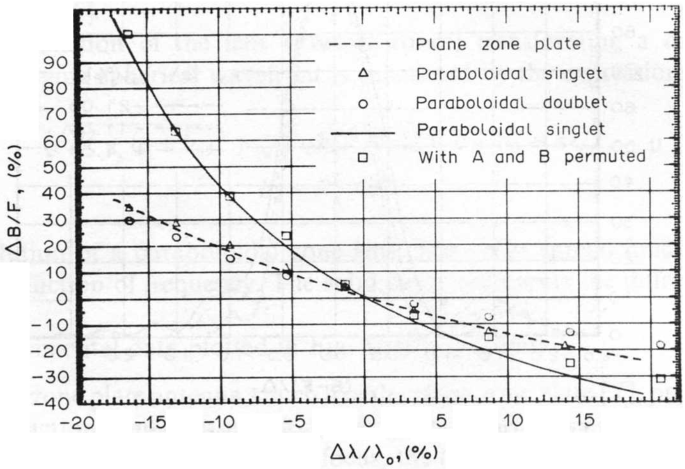

Fig. 4. Frequency characteristics of single and two-element zone lenses.

lobes is within 5–7% of the main lobe. The field of view of a paraboloidal singlet with $D_\lambda \sim 44$ is at least 30°.

The absence of reversibility for convex zone plates was proved experimentally. The lengths *A* and *B* were interchanged for a paraboloidal singlet and the intensity distribution was studied along the optical axis. In addition to the main maximum these experiments revealed a few auxiliary maxima of intensity comparable in amplitude with the main lobe. This redistribution of the energy incident on the zone lens into a few maxima reduces the efficiency of such devices. Shifting the source by $\Delta x \sim (2\text{–}3)\mathrm{R}_A$ away from the optical axis, where R_A is the Airy radius and a Lagrange invariant [8], causes the image to fall apart, that is the zone lens becomes inoperable. This is explained by the fact that the radii of Fresnel zones and the respective focal lengths get out of consistency at such displacements, i.e. the condition of tautochroism is violated.

TWO-COMPONENT PARABOLOIDAL ZONE LENS

Experimental studies of a multicomponent zone lens, developed by the present authors [9] on the base of Rayleigh–Wood zone plates, have indicated that such systems can have substantially improved information properties compared with their single-component counterparts. However, increasing the number of components in a zone lens may be criticized on the following grounds. Complexity associated with the lens size and labour involved in the alignment of the components increase appreciably. Losses of the power incident on the lens also increase as a consequence of multiple reflections between the components and absorption in the lens material. A sensible approach therefore would be to develop two-component zone lenses.

In our experiment, the two-component paraboloidal zone lens had the same aperture as its single-element counterpart, the front focal length of each component being $A/\lambda_0 = 47$. The spacing L between the components was chosen in agreement with the condition [9]

$$100\lambda \leqslant L \leqslant D^2/(0.61\lambda N),$$

where N is the number of elements resolved in an image row by the Raleigh criterion, and amounted to $L = 100\lambda$.

For each component of the lens, the Fresnel zone radii were determined assuming a plane incident wavefront. When this wave is incident on the convex face of the lens, the Fresnel zone radii may be computed as [4]

$$R_n = \sqrt{\frac{4b_0Fn\lambda + Fn^2/\lambda^2}{4F + n\lambda}}, \qquad R_n^2 = 4Fb_n,$$

where F is the focal length of the paraboloid, b_0 is the paraboloid vertex-to-focal-point distance along the optical axis, and b_n is the projected length of the nth Fresnel zone along the optical axis.

An analysis of lens performance revealed the following properties. For an axial source, the resolution (both along and across the optical axis) was identical with that of the single-component design. However, the field of view was expanded as compared to that of the singlet design, and amounted to at least 40°. Figure 4 shows that the frequency characteristic of the paraboloidal doublet coincides with the plot for the singlet.

CONCLUSION

Conclusions drawn upon the investigation into the properties of paraboloidal zone lens designs are as follows:

(i) use of curved surfaces in elements of computer (diffraction) optics expands the field of view and increases the number of imaged elements;

(ii) two component zone-lens designs offer a larger field of view than their singlet counterparts;

(iii) rotationally symmetric zone plates with surfaces generated by second-order curves retain their focusing properties in a wide range of the spectrum.

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